Stiffness and Strength of Asymmetric Laminates Made of Isotropic PCBA and PMMA Layers

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ABSTRACT: In this work the stiffness and strength of a composite material in the form of an asymmetric laminate are obtained from the properties of the constituent laminae made of isotropic layers, Lexan (PCBA) and Plexiglas (PMMA). The influence of the stacking sequence of the laminate and the interface between the laminae that affects the mechanical properties are investigated. A theoretical analysis based on the lamination theory was performed to determine the stress and strain distribution as well as the elastic constants. Experimental measurements with specimens made of asymmetric laminates of various stacking sequences were carried out. The obtained values were compared with the theoretical values given by the lamination theory and mechanics of materials approach. © 2006 Wiley Periodicals, Inc. J Appl Polym Sci 101: 4493–4503, 2006

Key words: laminate; stacking sequence; interface; lamination theory

INTRODUCTION

A laminate is a stack of laminae of isotropic or non isotropic materials. In recent years, laminated composite materials present great interest especially for lightweight constructions demanding high strength. Lamination is used to combine the best aspects of the constituent layers to achieve a more useful material. A major purpose of lamination is to tailor the directional dependence of strength and stiffness of a material to match the loading environment of the structural element. They are also made of isotropic layers and are usually known as sandwich materials. Laminated composites have been considered theoretically by many investigators. In Ref. 1, a theoretical solution was proposed for the case of a multilayered laminated composite beam under end load; Ref. 2 represents a more general solution than the previous one; Ref. 3 presents a plane stress solution applicable to a thin-walled cantilever beam with end load; Ref. 4 extended the preceeding result to include the influence of beam width for the Saint Venant solution to the bending of a sandwich beam. Lauterbach et al.[5] presented a finite element solution for Saint-Venant bending. Erdogan and Arin[6] considered cracked sandwich plates and performed a mathematical evaluation of the stress intensity factors. In Ref. 7, a study of the effect of thickness, stiffness, and the mass of the facings on the wave propagation and vibrations in an elastic symmetrical sandwich plate was carried out. In Ref. 8, the crack propagation in Lexan (PCBA) and Plexiglas (PMMA) sandwich plates was studied by using the method of dynamic caustics together with high-speed photography.

In the present work, the laminates used are made of isotropic plastic materials [Lexan (PCBA) and Plexiglas (PMMA)], forming asymmetric laminates. If multiple isotropic layers of various thicknesses are arranged symmetrically about a middle surface from both a geometric and a material property standpoint, the resulting laminate does not exhibit coupling between bending and extension. However, many physical applications of laminated composites require non-symmetrical laminates to achieve design requirements.

ANALYSIS

A composite laminate is made of two or more layers bonded together to act as a whole structural element. The stiffness of such a material depends on the properties of the layers. The material we deal is a laminate made of 2 and 4 layers of isotropic material.

In plane stress conditions in xyz axis system, the stress–strain relationships are given as:[9]

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11}Q_{12}0 \\ Q_{12}Q_{22}0 \\ 00Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (1)$$

The elements Q_{ij} of the stiffness matrix are related with the material properties as follows:

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$$Q_{11} = \frac{E}{1-\nu^2} = Q_{22},$$

$$Q_{12} = \frac{\nu E}{1-\nu^2}, Q_{66} = \frac{E}{2(1+\nu)} = G \quad (2)$$

Equation (1) can be thought of a stress–strain relationship for the kth layer of a multilayered laminate. Thus, it can be written as:

$$\{\sigma\}_k = [Q]_k\{\varepsilon\}_k \quad (3)$$

By substitution of the strain variation through the thickness in this relationship, the stresses in the kth layer can be expressed in terms of the laminate middle surface strains and curvatures as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \left\{ \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right\} \quad (4)$$

where $\varepsilon_x{}^0$, $\varepsilon_y{}^0$, $\gamma_{xy}{}^0$ and κ_x, κ_y, κ_{xy} are the middle surface strains and curvatures respectively, and z is the ordinate through the thickness of the laminate. The entire force and moment resultants for an N-layered laminate are defined as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = [A_{ij}] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [B_{ij}] \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (5)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = [B_{ij}] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [D_{ij}] \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (6)$$

where the matrices A_{ij}, B_{ij}, D_{ij} are given as:

$$A_{ij} = \sum_{k=1}^{N} (Q_{ij})_k (z_k - z_{k-1}),$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N} (Q_{ij})_k (z_k^2 - z_{k-1}^2),$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} (Q_{ij})_k (z_k^3 - z_{k-1}^3) \quad (7a,b,c)$$

In eq. (7), the A_{ij} are extensional stiffnesses, the B_{ij} are called coupling stiffnesses, and the D_{ij} are called bending stiffnesses. The presence of the B_{ij} implies coupling between bending and extension of a laminate. The above eqs. (5) and (6) can be written in a contracted form as:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{Bmatrix} A & B \\ B & D \end{Bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \text{ or } \begin{Bmatrix} N \\ M \end{Bmatrix} = [K] \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \quad (8)$$

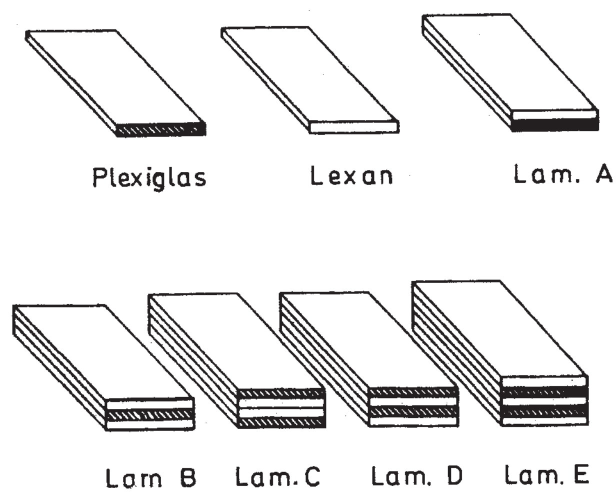

Figure 1 Schematic representation of the material used.

From this relationship, the middle surface strains and curvatures can be obtained by the matrix $[K]$ i.e.:

$$\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} = [F] \begin{Bmatrix} N \\ M \end{Bmatrix} \quad (9)$$

where:

$$[F] = [K]^{-1}$$

On the other hand, the elastic constants in the laminate plane E_x, E_y, V_{xy}, G_{xy} can be determined by the classical theory of laminated plates by the aid of models that assume uniform stress through the thickness of the laminate. Thus:

$$\frac{1}{E_x} = A'_{11}, \frac{1}{E_y} = A'_{22}, v_{xy} = \frac{-A'_{12}}{A'_{11}},$$

$$\frac{1}{G_{xy}} = A'_{66} \quad (10)$$

where $A'_{ij} = A_{ij}^{-1}$ and $A_{ij} = \frac{1}{t}\left[\sum_{k=1}^{N} (Q_{ij})_k t_k\right]$.

Here t_k is the thickness of kth layer and t the thickness of the whole laminate.

EXPERIMENTAL

Specimens were made of 2 or 4 layers in asymmetrical combination of Lexan (PCBA) and Plexiglas (PMMA) layers having 2×10^{-3} m thickness each, thus providing the form of a laminate designated as Lam. A and Lam. D from the whole series appearing in Figure 1. For the bonding of the layers, special glue (trichloroethylene- dicloromethane 2/1) was used. To measure the strains, longitudinal and transversal strain gauges

TABLE I
Elastic Modulus, Poisson Ratio, and Ultimate Stress of Lexan and Plexiglas

Material	E (GPa)	V	σ^{uft}.(MPa)
Plexiglas	3.19	0.33	61.2
Lexan	2.45	0.34	48.8

(KYOWA type with gauge length 2 mm) and Huggenberger extensometer were used.

Tensile experiments were carried out, to measure mechanical properties, using dogbone specimens with total thickness nominally varying from 4×10^{-3} to 8×10^{-3} m, according to ASTM D638. The width of the specimens was 3×10^{-2} m near the grips and varied from 12×10^{-3} to 19×10^{-3} m at the mid-length whereas the overall length varied from 180×10^{-3} to 260×10^{-3} m.

The experimentally obtained values of the Elastic modulus, E, Poisson ratio, v, and ultimate stress σ_{ult}. for PCBA and PMMA are given in Table I.

Theoretical calculations and results

Stresses, strains

The elements of the stiffness matrix Q_{ij} can be calculated for each material from eq. (2) using the values of E, v for Lexan and Plexiglas given in Table I. Thus:

$$[Q_{ij}]_L = \begin{bmatrix} 27702.40 & 9418.82 & 0 \\ 9418.82 & 27702.40 & 0 \\ 0 & 0 & 9141.79 \end{bmatrix} \times 10^5 \frac{N}{m^2}$$

$$[Q_{ij}]_P = \begin{bmatrix} 35798.45 & 11813.49 & 0 \\ 11813.49 & 35798.45 & 0 \\ 0 & 0 & 11992.48 \end{bmatrix} \times 10^5 \frac{N}{m^2} \quad (11a,b)$$

For the first series, laminate (A1), with two layers from eqs. (7):

$$A_{ij} = \{[Q_{ij}]_L + [Q_{ij}]_P\} h \left(\frac{N}{m}\right)$$

$$B_{ij} = \{[Q_{ij}]_P - [Q_{ij}]_L\} \frac{h^2}{2} (N)$$

$$D_{ij} = \{[Q_{ij}]_P + [Q_{ij}]_L\} \frac{h^3}{3} (Nm) \quad (12a,b,c)$$

where $h = t_L = t_P = 2 \times 10^{-3}$ m denotes the thickness of each layer.

The $[K]$ and $[F]$ matrices from eqs. (8) and (9) are found as:

$$[K] = \begin{bmatrix} 12700.17 & 4246.46 & 0 & & & \\ 4246.46 & 12700.17 & 0 & & & \\ 0 & 0 & 4226.85 & & & \\ 161.92 & 47.89 & 0 & 171.45 & 57.33 & 0 \\ 47.89 & 161.92 & 0 & 57.33 & 171.45 & 0 \\ 0 & 0 & 57.01 & 0 & 0 & 57.06 \end{bmatrix} \times 10^5 \quad (13)$$

and:

$$[F] = \begin{bmatrix} 9 & -3 & 0 & & & \\ -3 & 9 & 0 & & sym. & \\ 0 & 0 & 24 & & & \\ -9 & 3 & 0 & 665 & -223 & 0 \\ 3 & -9 & 0 & -223 & 665 & 0 \\ 0 & 0 & -24 & 0 & 0 & 1776 \end{bmatrix} \quad (14)$$

For laminate (A2) with inverse stacking sequence in a similar way we obtain:

$$A_{ij} = \{[Q_{ij}]_L + [Q_{ij}]_P\} h \left(\frac{N}{m}\right)$$

$$B_{ij} = \{[Q_{ij}]_L - [Q_{ij}]_P\} \frac{h^2}{2} (N)$$

$$D_{ij} = \{[Q_{ij}]_P + [Q_{ij}]_L\} \frac{h^3}{3} (Nm) \quad (15a,b,c)$$

$$[K] = \begin{bmatrix} 12700.17 & 4246.46 & 0 & & & \\ 4246.46 & 12700.17 & 0 & sym. & & \\ 0 & 0 & 4226.85 & & & \\ -161.92 & -47.89 & 0 & 171.45 & 57.33 & 0 \\ -47.89 & -161.92 & 0 & 57.33 & 171.45 & 0 \\ 0 & 0 & -57.01 & 0 & 0 & 57.06 \end{bmatrix} \times 10^5 \quad (16)$$

$$[F] = \begin{bmatrix} 9 & -3 & 0 & & & \\ -3 & 9 & 0 & & sym. & \\ 0 & 0 & 24 & & & \\ 9 & -3 & 0 & 665 & -223 & 0 \\ -3 & 9 & 0 & -223 & 665 & 0 \\ 0 & 0 & 24 & 0 & 0 & 1776 \end{bmatrix} \quad (17)$$

If we compare the matrix $[K]$ of this laminate with the previous one of Lam.(A1), we observe that there is change in the sign of some terms because of the inverse stacking sequence of the layers.

For laminate (D1), with four layers from eqs. (7):

$$A_{ij} = \{[Q_{ij}]_L + [Q_{ij}]_P\}2h \ (N/m)$$
$$B_{ij} = \{[Q_{ij}]_L - [Q_{ij}]_P\}h^2 \ (N)$$
$$D_{ij} = \{[Q_{ij}]_P + [Q_{ij}]_L\}\frac{8h^3}{3} \ (Nm) \quad (18a,b,c)$$

$$[K] = \begin{bmatrix} 25400.34 & 8492.92 & 0 & & & \\ 8492.92 & 25400.34 & 0 & & sym. & \\ 0 & 0 & 8453.71 & & & \\ -323.84 & -95.79 & 0 & 1352.57 & 452.25 & 0 \\ -95.79 & -323.84 & 0 & 452.25 & 1352.57 & 0 \\ 0 & 0 & -114.03 & 0 & 0 & 450.16 \end{bmatrix} \times 10^5 \quad (19)$$

We can observe that matrix $[B]$ is not zero because the laminate is not symmetric:

$$[F] = \begin{bmatrix} 4 & -1 & 0 & & & \\ -1 & 4 & 0 & sym. & & \\ 0 & 0 & 12 & & & \\ 1 & -0.4 & 0 & 84 & -28 & 0 \\ -0.4 & 1 & 0 & -28 & 84 & 0 \\ 0 & 0 & 3 & 0 & 0 & 223 \end{bmatrix} \quad (20)$$

For laminate (D2), with four layers, and different stacking sequence, from eqs. (7) we have:

$$A_{ij} = \{[Q_{ij}]_L + [Q_{ij}]_P\}2h \ (Nm)$$
$$B_{ij} = \{[Q_{ij}]_P - [Q_{ij}]_L\}h^{2} \ (N)$$
$$D_{ij} = \{[Q_{ij}]_P + [Q_{ij}]_L\}\frac{8h^{3}}{3} \ (Nm) \quad (21a,b,c)$$

$$[K] = \begin{bmatrix} 25400.34 & 8492.92 & 0 & & & \\ 8492.92 & 25400.34 & 0 & & sym. & \\ 0 & 0 & 8453.71 & & & \\ 323.84 & 95.79 & 0 & 1352.57 & 452.25 & 0 \\ 95.79 & 323.84 & 0 & 452.25 & 1352.57 & 0 \\ 0 & 0 & 114.03 & 0 & 0 & 450.16 \end{bmatrix} \times 10^5 \quad (22)$$

The difference of this matrix with respect to the previous one of Lam.(D1) is that the elements of matrix $[B]$ have the opposite sign.

$$[F] = \begin{bmatrix} 4 & -1 & 0 & & & \\ -1 & 4 & 0 & sym. & & \\ 0 & 0 & 12 & & & \\ -1 & 0.4 & 0 & 84 & -28 & 0 \\ 0.4 & -1 & 0 & -28 & 84 & 0 \\ 0 & 0 & -3 & 0 & 0 & 223 \end{bmatrix} \quad (23)$$

To proceed to the calculation of the stresses of the material, let us take into account eq. (4). The strains can be calculated by taking under consideration eq. (5) and that:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (24)$$

Now, for asymmetric cases from eq. (9) for uniaxial tension i.e., $N_x = N$, $N_y = N_{xy} = M_x = M_y = M_{xy} = 0$:

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} A'_{11} \\ A'_{21} \\ 0 \\ B'_{11} \\ B'_{21} \\ 0 \end{Bmatrix} N_x \quad (25)$$

Taking into account eq. (24), we have:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} A'_{11} \\ A'_{12} \\ 0 \end{Bmatrix} N_x + z \begin{Bmatrix} B'_{11} \\ B'_{12} \\ 0 \end{Bmatrix} N_x \quad (26)$$

By the aid of eq. (4), we finally obtain:

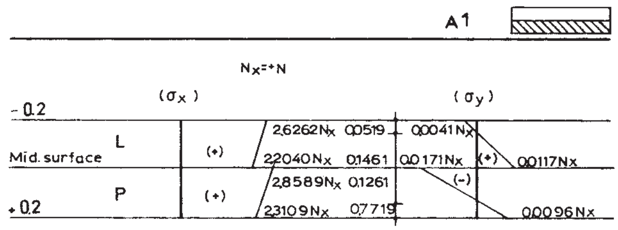

Figure 2 Variation of laminate stresses σ_x, σ_y in Lam.A1.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{bmatrix} A'_{11} \\ A'_{12} \\ 0 \end{bmatrix} + \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{bmatrix} B'_{11} \\ B'_{12} \\ 0 \end{bmatrix} z) \, N_x \quad (27)$$

For the first series laminate (A1), by the aid of eq. (2) and from eq. (14), we have:

In Lexan: $\sigma_x = (2.2040 - 2.1112z)N_x$, $\sigma_y = (0.0117 + 0.0792z)N_x$, $\tau_{xy} = 0$

In Plexiglas: $\sigma_x = (2.8589 - 2.7399z)N_x$, $\sigma_y = (-0.0171 + 0.1337z)N_x$, $\tau_{xy} = 0$

The variation of stresses is illustrated in Figure 2.

For laminate (A2) in a similar way and by the aid of eq. (17), we obtain:

In Lexan: $\sigma_x = (2.2040 + 2.1112z)N_x$, $\sigma_y = (0.0117 - 0.0792z)N_x$, $\tau_{xy} = 0$

In Plexiglas: $\sigma_x = (2.8589 + 2.7399z)N_x$, $\sigma_v = (-0.0171 + 0.1337z)N_x$, $\tau_{xy} = 0$.

The variation is illustrated in Figure 3.

For laminate (D1) by the aid of eq. (29), we have:

In Lexan: $\sigma_x = (1.0918 + 0.2651z)N_x$, $\sigma_y = (0.0066 - 0.0097z)N_x$, $\tau_{xy} = 0$

In Plexiglas: $\sigma_x = (1.4161 + 0.3441z)N_x$, $\sigma_v = (-0.0073 - 0.0165z)N_x$, $\tau_{xy} = 0$

The variation is illustrated in Figure 4.

Finally, for laminate (D2) by the aid of eq. (32), we obtain:

In Lexan: $\sigma_x = (1.0918 - 0.2651z)N_x$, $\sigma_y = (0.0066 + 0.0097z)N_x$, $\tau_{xy} = 0$

In Plexiglas: $\sigma_x = (1.4161 - 0.3441z)N_x$, $\sigma_y = (-0.0073 + 0.0165z)N_x$, $\tau_{xy} = 0$

The variation is illustrated in Figure 5.

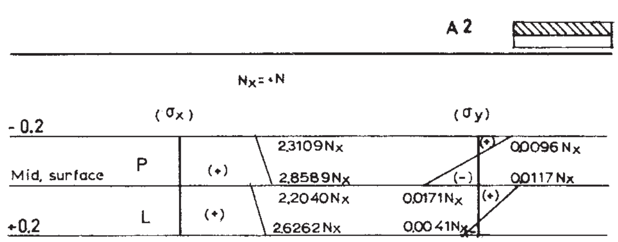

Figure 3 Variation of laminate stresses σ_x, σ_y in Lam.A2.

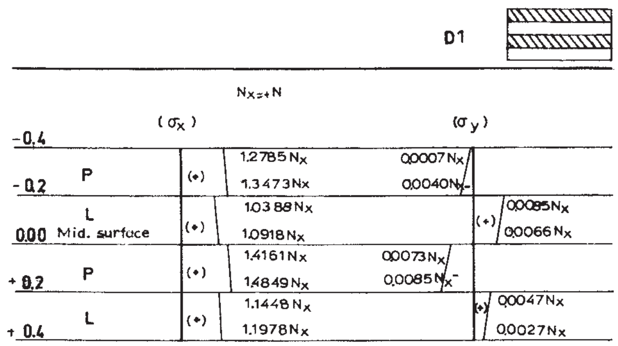

Figure 4 Variation of laminate stresses σ_x, σ_y in Lam.D1.

Next, the strains for the case of uniaxial tension will be calculated. To make comparison in each case the strains for equivalent Lexan and Plexiglas material are also given. For each laminate and for N_x force per unit width in (N/m) or (kp/cm), if we use Lexan with total thickness $t = nh = n \times 0.2$ when n is the number of layers and from Table I: $E_L = 2.45$ GPa $= 24{,}500$ kp/cm^2.

$$\varepsilon_x = \frac{\sigma_x}{E_L} = \frac{N_x / bt}{E_L} = \frac{N_x}{nh \times 24500}$$

$$\varepsilon_y = -\nu\varepsilon_x = -0.34\varepsilon_x$$

If we use Plexiglas with $t = nh = n \times 0.2$ and from Table I: $E_P = 3.19$ GPa $= 31{,}900$ Kp/cm^2 we have:

$$\varepsilon_x = \frac{\sigma_x}{E_p} = \frac{N_x / bt}{E_p} = \frac{N_x}{nh \times 31900}$$

$$\varepsilon_y = -\nu\varepsilon_x = -0.33\varepsilon_x$$

Now, for asymmetric laminates from eq. (26), we can observe that there are two terms in the laminate strains because of asymmetry. Thus for laminate (A1) with $t = 2h = 2 \times 0.2 = 0.4$ cm, we have:

In Lexan: $\varepsilon_x = 10.2 \times 10^{-5} N_x$, $\varepsilon_y = -3.5 \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

In Plexiglas: $\varepsilon_x = 7.8 \times 10^{-5}\, N_x$, $\varepsilon_y = -2.6 \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

In the Laminate: $\varepsilon_x = (9 - 9z) \times 10^{-5}\, N_x$, $\varepsilon_y = (-3 + 3z) \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

The variation of strains is illustrated in Figure 6. Similarly for laminate (A2) we obtain:

In Lexan: $\varepsilon_x = 10.2 \times 10^{-5}\, N_x$, $\varepsilon_y = -3.5 \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

In Plexiglas: $\varepsilon_x = 7.8 \times 10^{-5}\, N_x$, $\varepsilon_y = -2.6 \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

In the Laminate: $\varepsilon_x = (9 + 9z) \times 10 - 5\, N_x$, $\varepsilon_y = (-3 - 3z) \times 10^{-5}\, N_x$, $Y_{xy} = 0$

The variation is illustrated in Figure 7.

For laminate (D1) with $t = 4h = 4 \times 0.2 = 0.8$ cm we obtain:

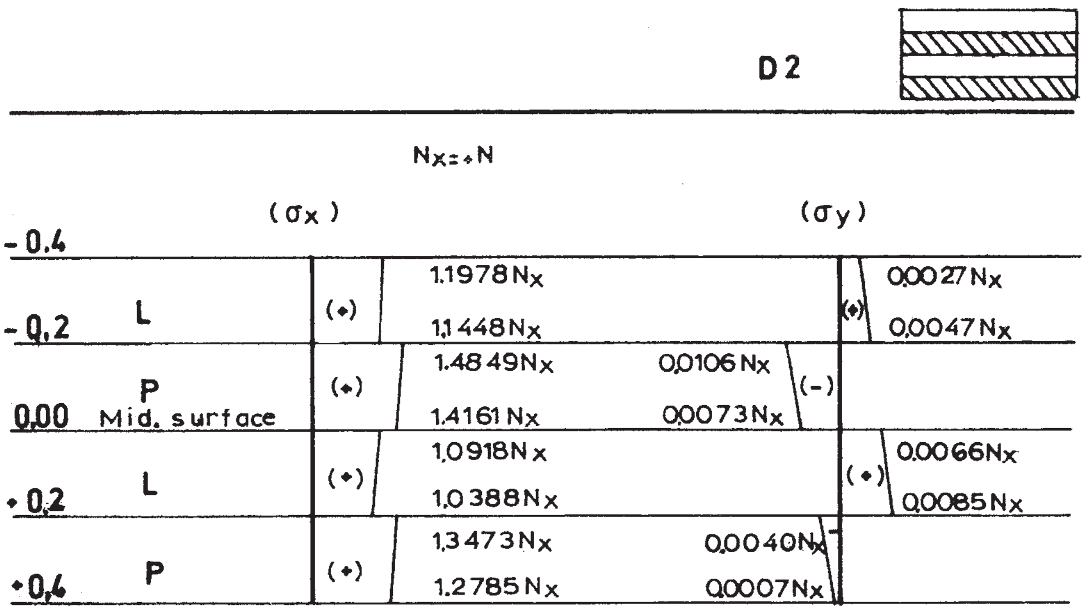

Figure 5 Variation of laminate stresses σ_x, σ_y in Lam.D2.

In Lexan: $\varepsilon_x = 5.1 \times 10 - 5\, N_x$, $\varepsilon_y = -1.7 \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

In Plexiglas: $\varepsilon_x = 3.9 \times 10^{-5}\, N_x$, $\varepsilon_y = -1 \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

In the Laminate: $\varepsilon_x = (4 + z) \times 10^{-5}\, N_x$, $\varepsilon_y = (-1 - 0.4z) \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

The variation is illustrated in Figure 8.

Finally, for laminate (D2) we obtain:

In Lexan: $\varepsilon_x = 5.1 \times 10^{-5}\, N_x$, $\varepsilon_y = -1.7 \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

In Plexiglas: $\varepsilon_x = 3.9 \times 10^{-5}\, N_x$, $\varepsilon_y = -1 \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

In the Laminate: $\varepsilon_x = (4 - z) \times 10^{-5}\, N_x$, $\varepsilon_y = (-1 + 0.4z) \times 10^{-5}\, N_x$, $\gamma_{xy} = 0$

The variation is illustrated in Figure 9.

Elastic constants

Let us now calculate the elastic constants of the different laminates used. In this analysis, the laminate is considered to be made of a layer of a homogeneous material. The calculations will be carried out through eqs. (10) and through the following approximate formulae of the rule of mixtures.

$$E_c = E_L U_L + E_p U_p \tag{28}$$

$$v_c = v_L U_L + v_p U_p \tag{29}$$

where E_L, v_L, U_L denote the elastic modulus, Poisson ratio, and volume fraction of the Lexan and E_p, v_p, U_p those of the Plexiglas, respectively. The volume fractions are given as the ratios of the volume of each material to the total volume of the laminate and with $U_L + U_p = 1$. Using the values given in Table I for E and v and the values of $[K]$ or $[F]$ matrix for A_{ij}, B_{ij} and D_{ij} given in eqs. (13), (14), (16), (17), (19), (20), and (23) by the aid of eq. (10) and eq. (2), the values of the laminate elastic modulus and Poisson ratio can be obtained. These values appear in Table II and Table III, respectively. It can be observed that there is a very good coincidence between the values of Poisson ratio calculated by the formulae of laminate theory given in eq. (10) and those calculated by the approximate theory of the rule of mixtures given in eq. (29). Also, it can be said that the stacking sequence of the laminate almost does not influence the Poisson ratio whereas it influences slightly the elastic modulus.

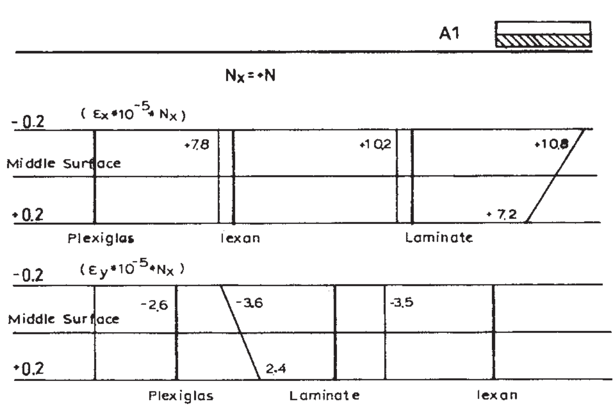

Figure 6 Variation of laminate strains ε_x, ε_y in Lam.A1.

Determination of the ultimate load carrying capacity

It is required to determine the ultimate load carrying capacity of a laminate, defined as one consisting of two or more dissimilar materials, under a tensile load

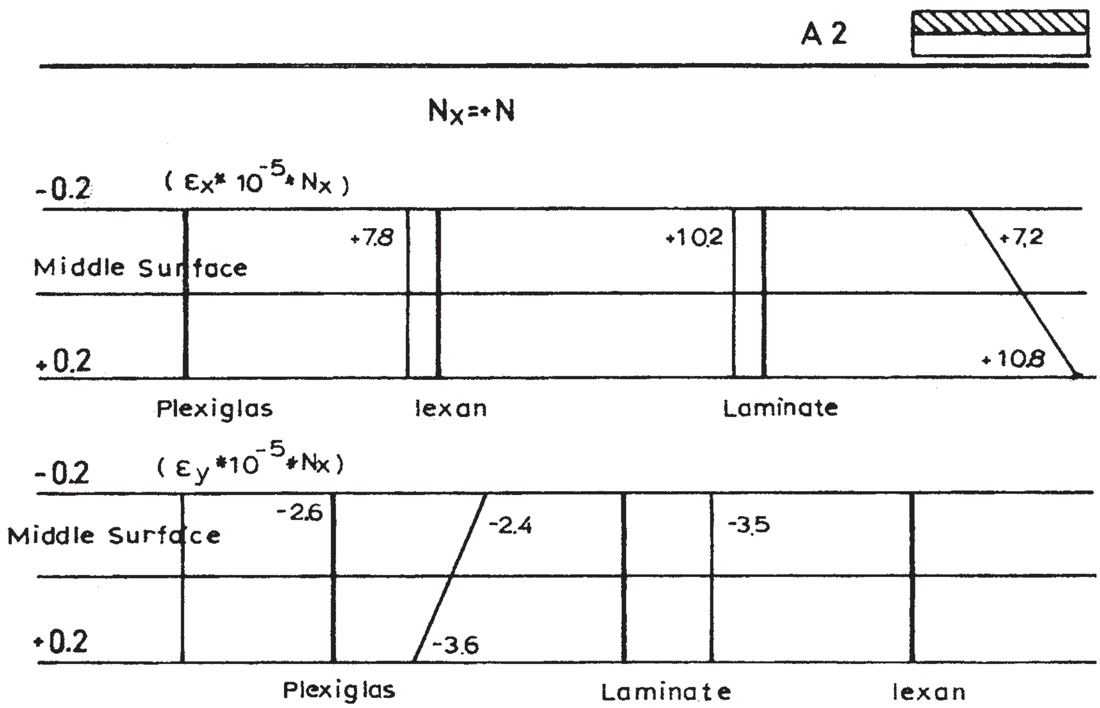

Figure 7 Variation of laminate strains ε_x, ε_y in Lam.A2.

P. The laminate is composed of 2 or 4 layers of Lexan/Plexiglas with thickness 2 mm for each layer as mentioned earlier.

Since strains in all layers at a particular cross section are equal:

$$\varepsilon_L = \varepsilon_P \rightarrow \frac{\sigma_L}{E_L} = \frac{\sigma_P}{E_P} \tag{30}$$

which yields:

$$\sigma_L = \frac{E_L}{E_P}\sigma_P \text{ or } \sigma_P = \frac{E_P}{E_L}\sigma_L \tag{31}$$

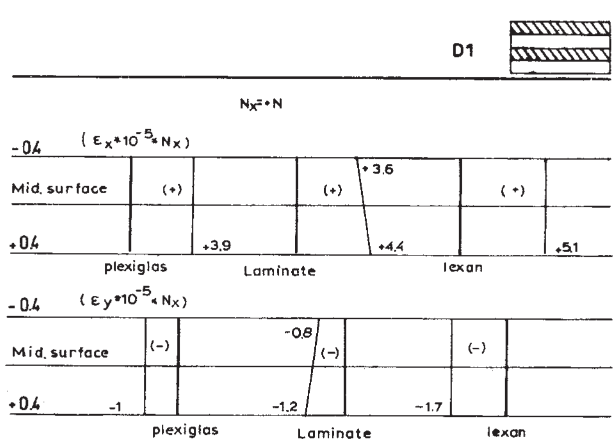

Figure 8 Variation of laminate strains ε_x, ε_y in Lam.D1.

From these relationships by using the values of Table I we have:

$$\text{(a) If } \sigma_c^{\text{ult}} = \sigma_P^{\text{ult}} = 61.2\,MPa \rightarrow \sigma_L = 47\,MPa < \sigma_L^{\text{ult}} \tag{32}$$

$$\text{(b) If } \sigma_c^{\text{ult}} = \sigma_L^{\text{ult}} = 48.8\,MPa \rightarrow \sigma_P = 63.54\,MPa > \sigma_P^{\text{ult}} \tag{33}$$

The subscripts *c*, *L*, *P* denote the composite, the Lexan, and the Plexiglas, respectively. The first relationship states that when the ultimate stress for Plexiglas is reached the stress value in Lexan is less than its ultimate, whereas the second relationship states that when the ultimate stress for Lexan is reached the stress value in Plexiglas has been exceeded.

Therefore, the criteria of failure of the laminate is Plexiglas and the mean ultimate load carrying capacity of the material is:

$$\text{(1) For series A: } P = \sigma_P^{\text{ult}} \times b_p \times t_p + \sigma_L \times b_L \times t_L \tag{34}$$

where b_P and b_L denote the width and t_P and t_L denote the thickness of the Lexan and Plexiglas, respectively, which are equal to *h*. The above relationship, by using eqs. (31)–(33) given that the thickness of each layer is 2 mm and the nominal width varies from 12 to 19 mm

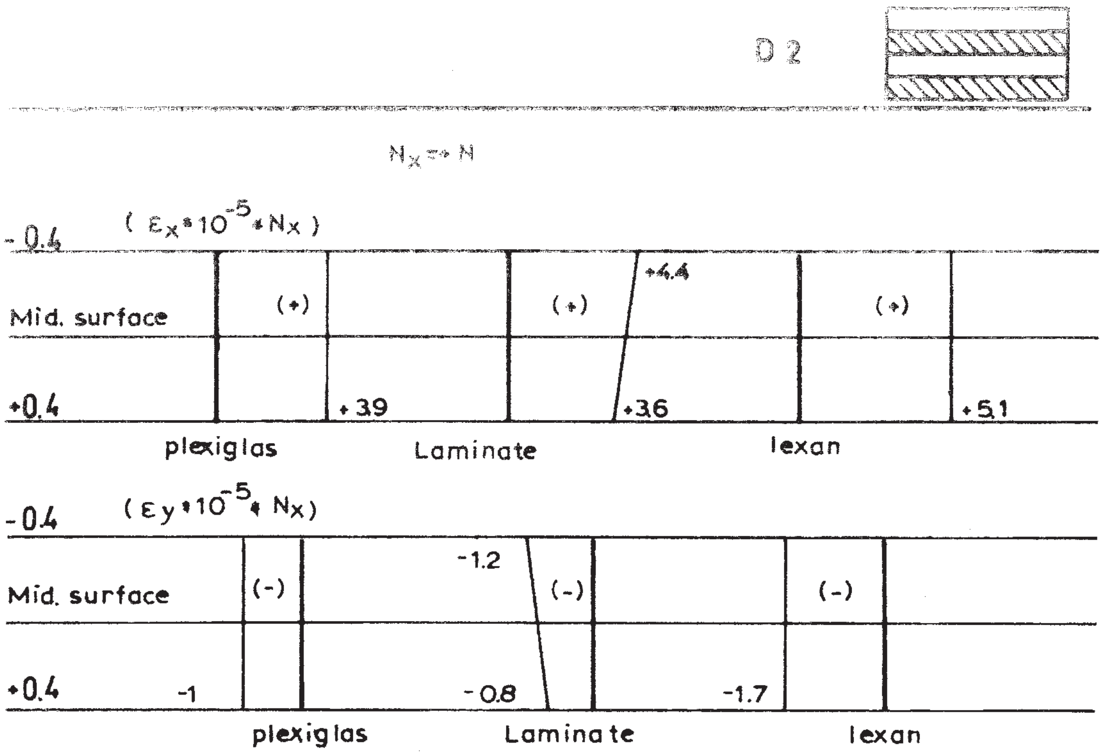

Figure 9 Variation of laminate strains ε_x, ε_y in Lam.D2.

depending on each series (in this one is 12 mm), yields:

$$P = (61.2 \times 10^6 \times 12.38 \times 10^{-3} \times 2 \times 10^{-3} + 47 \times 10^6 \times 12.38 \times 10^{-3} \times 2 \times 10^{-3}) = 2679.1N$$

A check should be made of the ultimate load on the laminate after the failure of Plexiglas. In this case, the ultimate load that the material is able to support is:

$$P' = 48.8 \times 10^6 \times 12.27 \times 10^{-3} \times 2 \times 10^{-3} = 1197.6N \quad (35)$$

(2) For series D: $P = \sigma_P^{\mathrm{ult}} \times b_P \times \mathrm{t_P} \times 2 + \sigma_L \times b_L \times t_L \times 2$

Similarly, the above relationship given that the thickness of each layer is 2 mm and the normaly mean width is 18 mm yields:

$$P' = (61.2 \times 10^6 \times 18.01 \times 10^{-3} \times 2 \times 10^{-3} \times 2 + 47 \times 10^6 \times 18.01 \times 10^{-3} \times 2 \times 10 \times 2) = 7794.73N$$

The ultimate load that the material is able to support is:

$$P' = 48.8 \times 10^6 \times 18.01 \times 10^{-3} \times 2 \times 10^{-3} \times 2 = 3515.6N$$

EXPERIMENTAL RESULTS

Figure 10 illustrates the initial part of the stress–strain diagram for Lexan and Plexiglas as derived from tensile experiments by using mechanical gauges (Hüggenberger) for the determination of the longitudinal strain. From these diagrams, the elastic moduli of Lexan and Plexiglas were evaluated as 2.45 and 3.19 GPa, respectively. Both the diagrams show a strong linear behavior for the variation of the stress versus strain for the two materials. It can be observed that

TABLE II
Theoretical Values of the Laminate Elastic Modulus

Laminate	E_c (GPa)	$(E_c)_{\mathrm{approx}}$ (GPa)
A1	2.784	2.820
D1	2.811	2.820

TABLE III
Theoretical Values of the Laminate Poisson Ratio

Laminate	v_c	$(v_c)_{\mathrm{app}}$
A1	0.335	0.335
D1	0.335	0.335

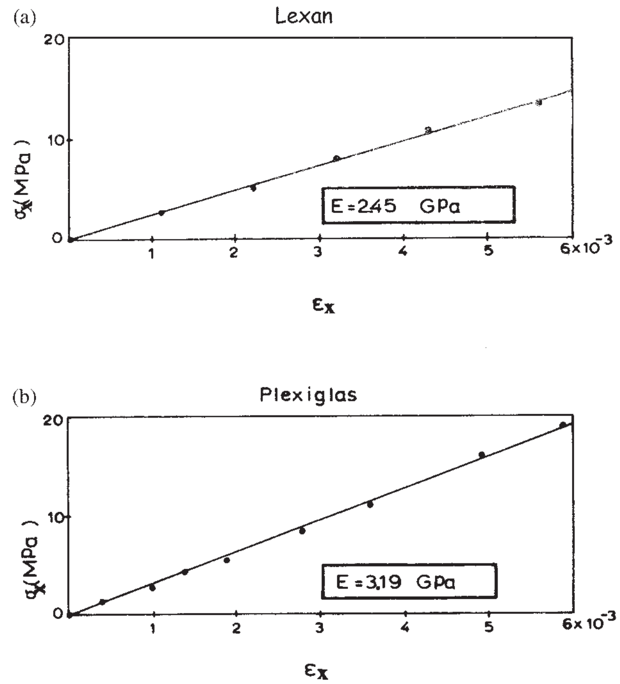

Figure 10 Variation of stress σ_x versus strain ε_x for Lexan and Plexiglas is derived from tensile experiments using mechanical gauges.

Lexan is a ductile material and its cross section decreases until the rupture of the material. Its failure almost coincides with its yielding and it is a material that it can be "trusted" when working in large strains. On the contrary, Plexiglas looks like a brittle material but its strength is better than Lexan. As a conclusion, it can be said that from the combination of these two materials and through different stacking sequences a mean behavior can be expected. In this, Lexan normally will contribute by its ductility and Plexiglas by its strength.

Figures 11 and 12 present the initial part of the stress–strain diagram for the series A1 and D1, respectively, as obtained from tensile experiments by using also mechanical gauges for the determination of the longitudinal strain. From these diagrams, the elastic moduli for the two materials were evaluated as 3.17

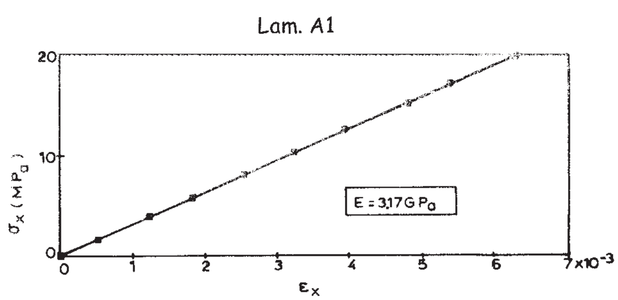

Figure 11 Variation of stress σ_x versus strain ε_x for Lam.A1 derived from tensile experiments using mechanical gauges.

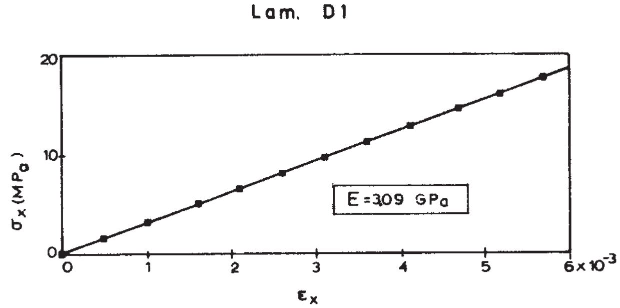

Figure 12 Variation of stress σ_x versus strain ε_x for Lam.D1 is derived from tensile experiments using mechanical gauges.

and 3.09 GPa, respectively. Again, in all diagrams, it can be observed that the variation of the stress versus strain shows a strong linear behavior. The variation of stress versus strain for the series A1 and D1 where the percentage of Plexiglas is 1/2 as obtained from tensile experiments by using electrical strain-gauges is illustrated in Figures. 13 and 14. The initial part of the diagrams served for the evaluation of the elastic modulus appears in Figures 13(a) and 14(a). The mean values for ultimate stress, elastic modulus, and Poisson ratio obtained experimentally are presented in Table IV.

It can be observed that Lam. D1 has higher strength than Lam. A1, although the percentage of Plexiglas, which has higher strength than Lexan, is the same. On the other hand, Lam. A1 presents higher elastic modulus than Lam. D1. The discrepancy between the two

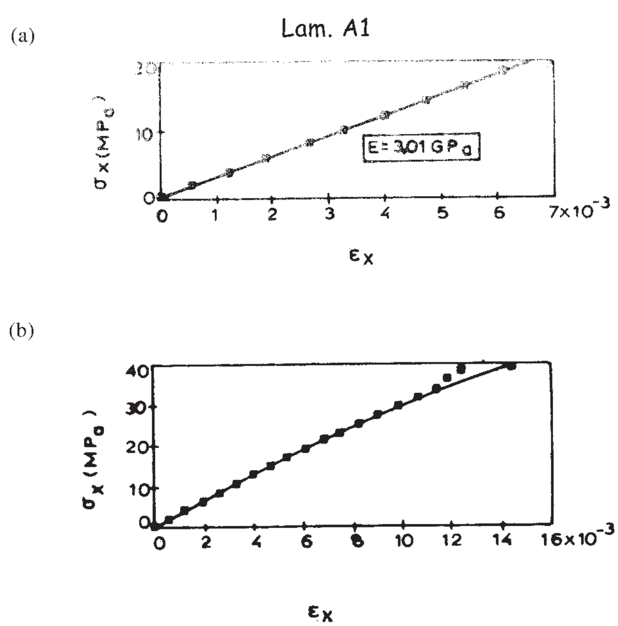

Figure 13 Variation of stress σ_x versus strain ε_x for Lam.A1 is derived from tensile experiments using strain-gauges. (a) Initial part and (b) entire curve.

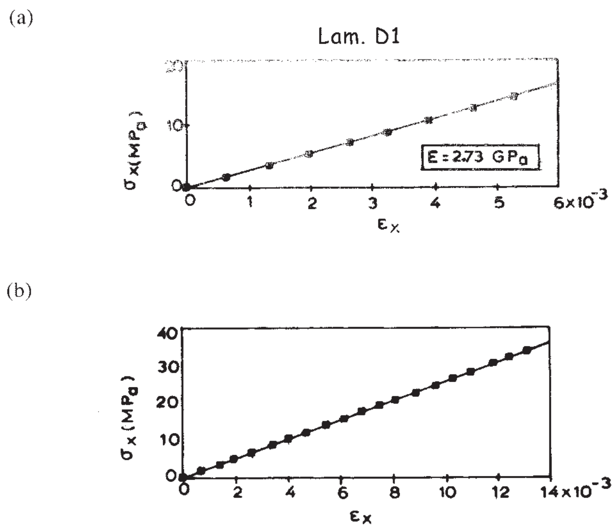

Figure 14 Variation of stress σ_x versus strain ε_x for Lam.D1 is derived from tensile experiments using strain-gauges. (a) Initial part and (b) entire curve.

TABLE V
Experimental and "Theoretical" Ultimate Load Carrying Capacity of the Laminates Used

Material	P_{exp} (N)	P (N)
Lam. A1	2526.7	2679.10
Lam. D1	7875.1	7794.73

types of measurements can be attributed to the difference between the two types of gauges. It is worth mentioning that the elastic moduli obtained through mechanical gauges are higher than those obtained through electrical strain-gauges.

The experimental Poisson ratio, v_c, of the laminates was determined by the ratio of the transversal strain to the longitudinal strain. The mean values are presented in Table IV. It can be observed that both laminates have the same Poisson ratio. On the other hand, it can be said that there is a discrepancy between the experimental results obtained and the theoretical values of Table III for the two series of laminates and that in all cases experimental values are superior to the theoretical ones.

As to the failure of the laminates, a general observation concerning the fracture mechanism can be stated: When the applied load increases, the bonding at the interface of the layers seems to weaken and failure occurs in lines lying at planes perpendicular to the loading direction. The phenomenon starts from the neck of the specimen and moves up and down toward the grips. It can observed that this phenomenon progresses even in the grips. This, continues up to whitening covers the specimen. The fracture surface is almost plane and perpendicular to the specimen axis, which means that the failure occurred only from normal stresses. After the fracture of the specimens, an effort was made to separate the layers of the materials from each other something that was impossible. The layers continued to be strongly bonded fact that led to the conclusion that the cracks started in a material rather than in an interface. This material probably was Plexiglas, which as a less ductile material cannot follow the large deformations of Lexan. This hypothesis is reinforced by the fact that the more was the percentage of Lexan the more "whitening" appeared before fracture and the more was the final deformation.

Finally, if we compare the ultimate load carrying capacity of the laminates under a tensile load P as derived from eqs. (34) and (35) with experimental results, [see Table V], we can observe that there is a discrepancy between the "theoretical" values and the mean experimental ones and that the experimental results are larger in all cases. This means that the construction of the laminates examined was fairly good.

Table VI is a comparative table for E_c and σ_c^{ult}, which includes the content of Lexan for symmetric and asymmetric laminates as obtained from this work and from Ref. 10.

It can be observed that Lam. B1 presents the highest strength, although the percentage of Plexiglas, which has higher strength than Lexan, is the lowest of the series. On the other hand, Lam. A1 presents the highest elastic modulus that cannot be considered reasonable since Lam. C1 has the highest percentage of Plexiglas the elastic modulus of which is higher than lexan. Normally, the laminate with higher percentage of

TABLE IV
Experimental Values of the Elastic Modulus, Poisson Ratio, and Ultimate Stress of the Materials Used

	Experimental modulus			
Material	E_c (GPa) (Hüggenberger)	E_c (GPa) (strain-gauge)	Exp. Poisson ratio v_c	Exp. ult. stress $\sigma_c^{ult} = \sigma_F$ (MPa)
Lexan	2.45		0.34	48.8
Plexiglas	3.19		0.33	61.2
Lam. A1	3.17	3.01	0.37	50.5
Lam. D1	3.09	2.73	0.37	53.4

TABLE VI
Comparative Table for Symmetric and Asymmetric Laminates

	Series	Elastic modulus Experimental modulus E_c (GPa)		$\sigma_c^{ult} = \sigma_F$ (MPa)	U_L	Theoretical modulus E_c (GPa)
		Huggenberger	Strain-gauge			
1	Lexan	2.45		2.450	48.8	1.00
2	Plexiglas	3.19		3.190	61.2	0.00
3	A1	3.17	3.01	2.784	50.5	0.50
4	B1	3.01	2.85	2.697	60.1	0.67
5	B2			2.943		0.33
6	C1	3.10	2.82	2.820	57.8	0.50
7	D1	3.09	2.73	2.811	53.4	0.50
8	E1	2.87	2.79	2.746	58.0	0.60
9	E2			2.894		0.40

Plexiglas should have higher elastic modulus as it can be seen in Table II in the theoretical values of elastic modulus, which were evaluated approximately as 2.78, 2.70, 2.82, 2.81, and 2.75 GPa for laminates A1, B1, C1, D1, E1, respectively, fact that it is not true in the experimental results where Lam. A1 has the highest elastic modulus from the measurements received through strain-gauges. Thus, the ranking for the theoretical elastic modulus for the various laminates is: B1, E1, A1, D1, C1, E2, B2 whereas for the Poisson ratio is B2, E2, C1, A1, D1, E1, B1, which is the opposite. Thus, it can be said that the laminates with three layers constitute the maximum and the minimum for the elastic constants.

From the measurements received through mechanical strain-gauges, the elastic modulus of Lam. A1 is the highest that is in accordance with the theoretical values. The discrepancy can be attributed again to the difference between the two types of gauges. It is worth mentioning that the elastic moduli obtained through mechanical gauges are higher than those obtained through electrical strain-gauges as mentioned previously.

CONCLUSIONS

From the comparison of theoretical and experimental results of the different types of laminates we conclude:

1. The appropriate combination of the layers and the position of each one in the laminate can have as result the increase of the strength.
2. Depending on the position of each layer and the combination in the laminate, an increase in the experimental values of the elastic modulus and Poisson ratio appears compared with the respective theoretical ones.
3. The elastic modulus of all types of the laminates examined remains between the values of the pure Lexan and pure Plexiglas i.e., $E_{Lex} < E_{Lam} < E_{P1}$.
4. The mean experimental values for the ultimate load are greater than the "theoretical" ones but the difference is not too considerable.

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